UiO : University of Oslo

Methods for interpolation of 3D data by smooth surfaces

(with a focus on different types of splines, their advantages and disadvantages)

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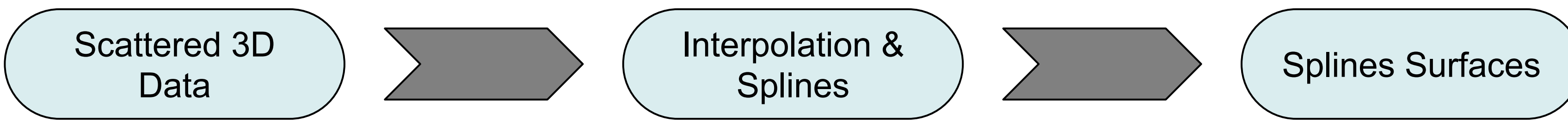
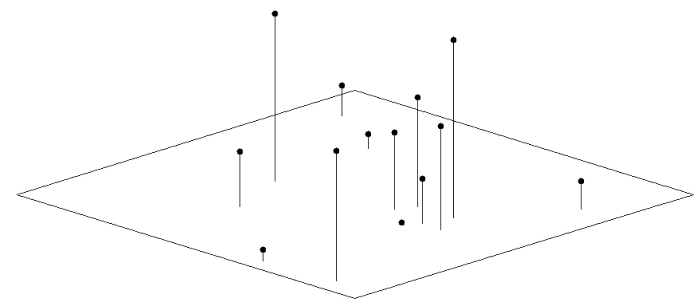
2021-06-09


Scattered 3D Data
Interpolation & Splines
Splines Surfaces



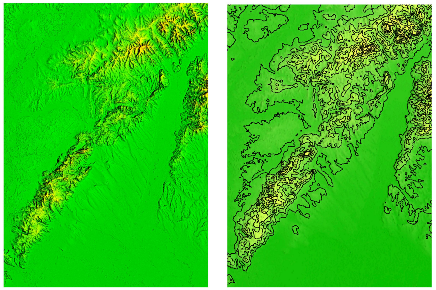
[LWS97] Figure 12a

Smooth surface interpolation of

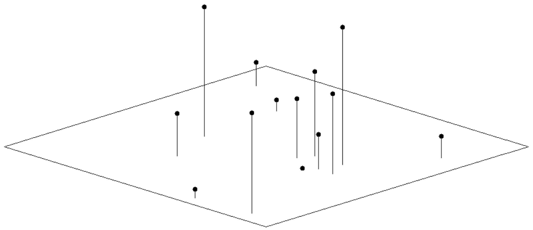
SCATTERED 3D DATA

[LWS97] “Scattered data interpolation with multilevel b-splines”, Lee, Wolberg, Shin, 1997

Scattered 3D data

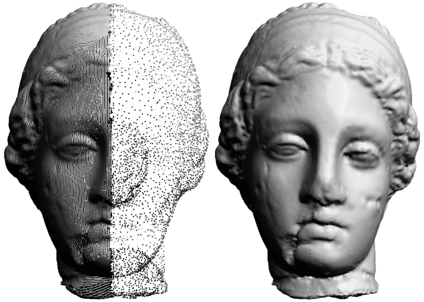


[Boh17] Figure 4



[LWS97] Figure 12a

[MP17] Figure 3&4

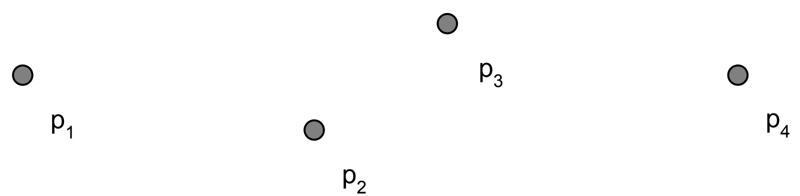


[OBS00] Figure 10

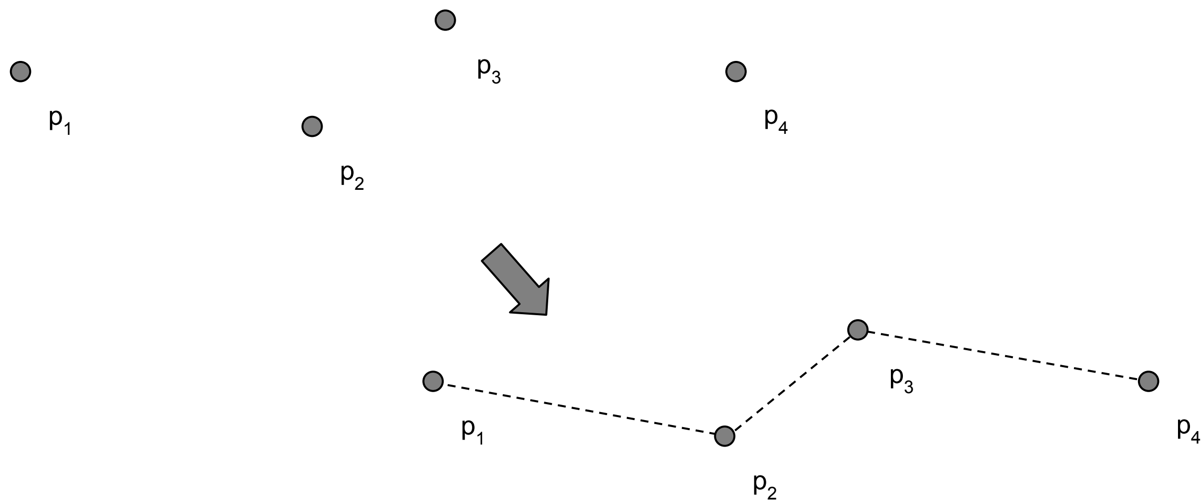
[Boh17] “Improvement of some interpolation methods for terrain reconstruction from scattered data”, Bohdal, 2017
[LWS97] “Scattered data interpolation with multilevel b-splines”, Lee, Wolberg, Shin, 1997
[MP17] “Comparison of mesh generated algorithms for railroad reconstruction”, Masson, Petry, 2017
[OBS00] “A multi-scale approach to 3d scattred data interpolation with compactly supported basis functions”, Ohtake, Belyaev, Seide, 2000

INTERPOLATION

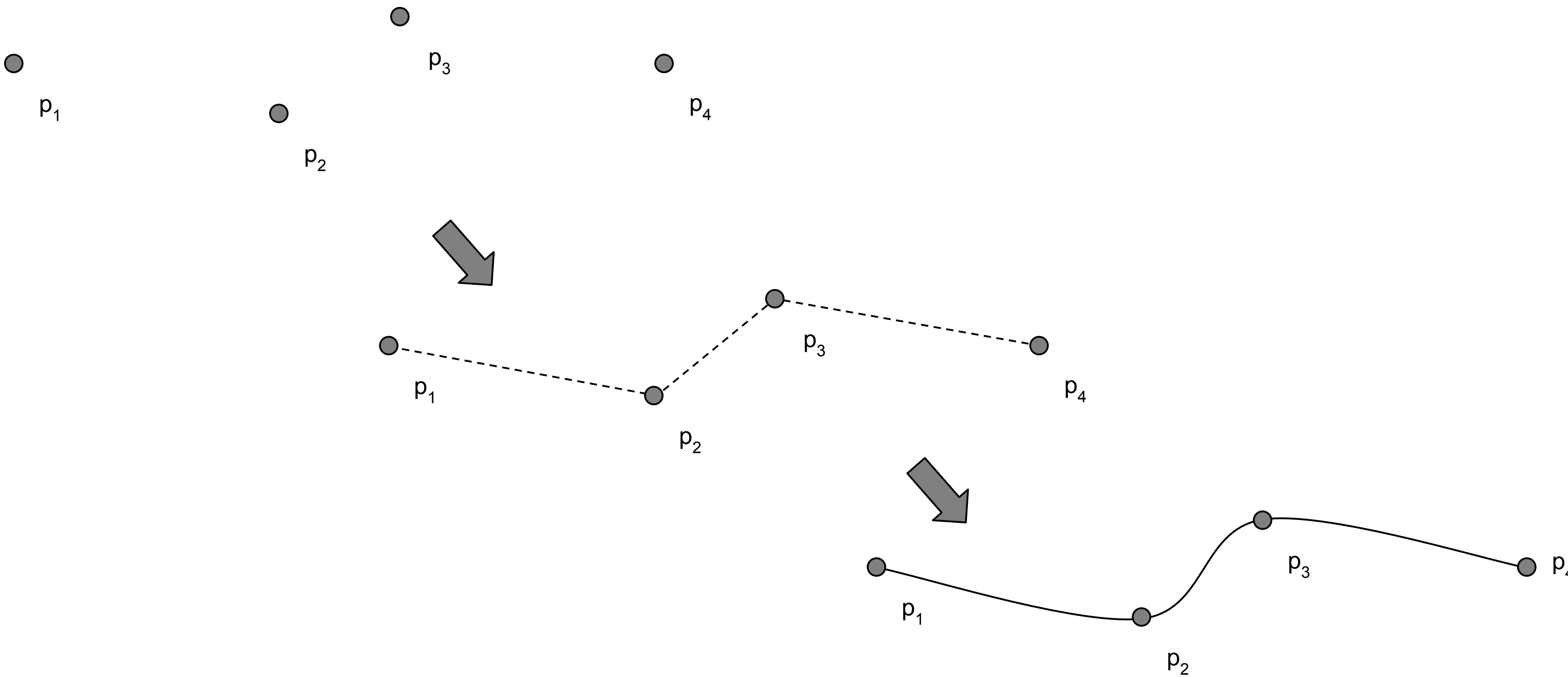
Interpolation



Interpolation

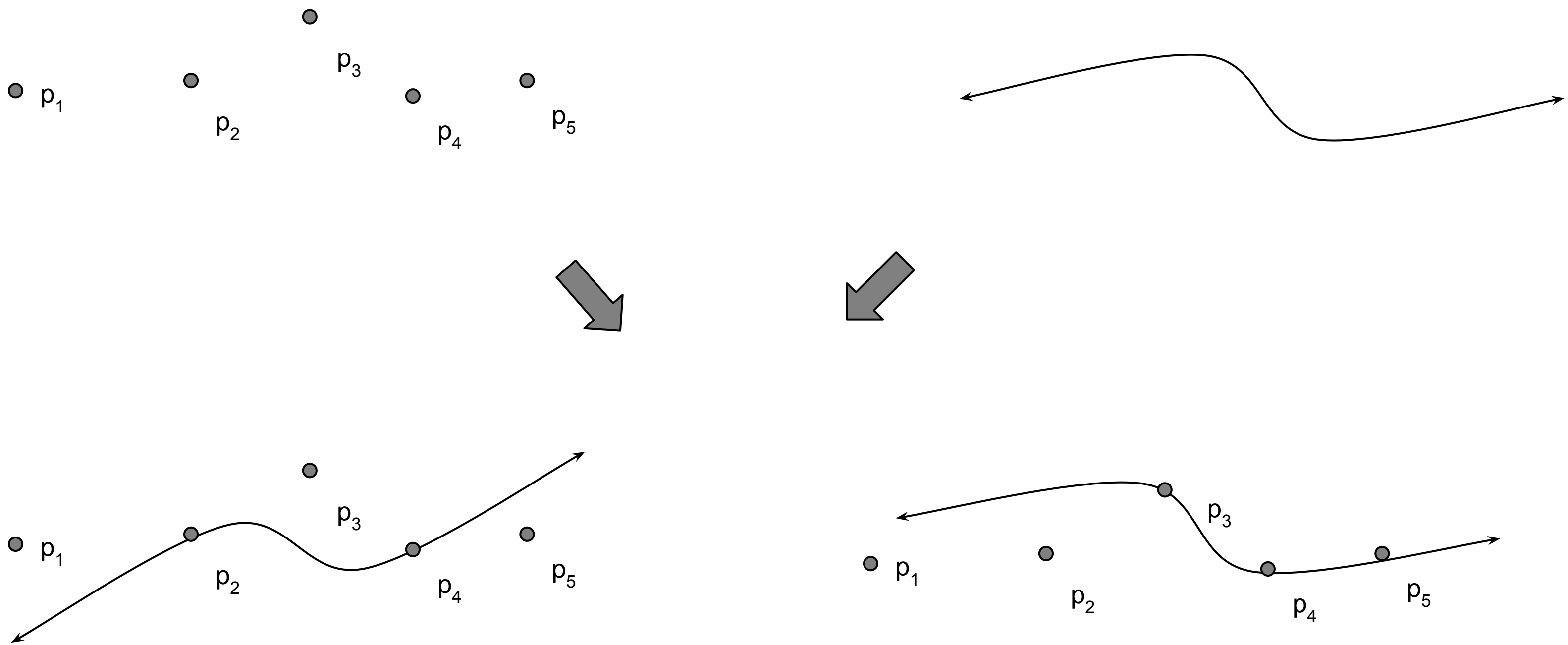


Interpolation

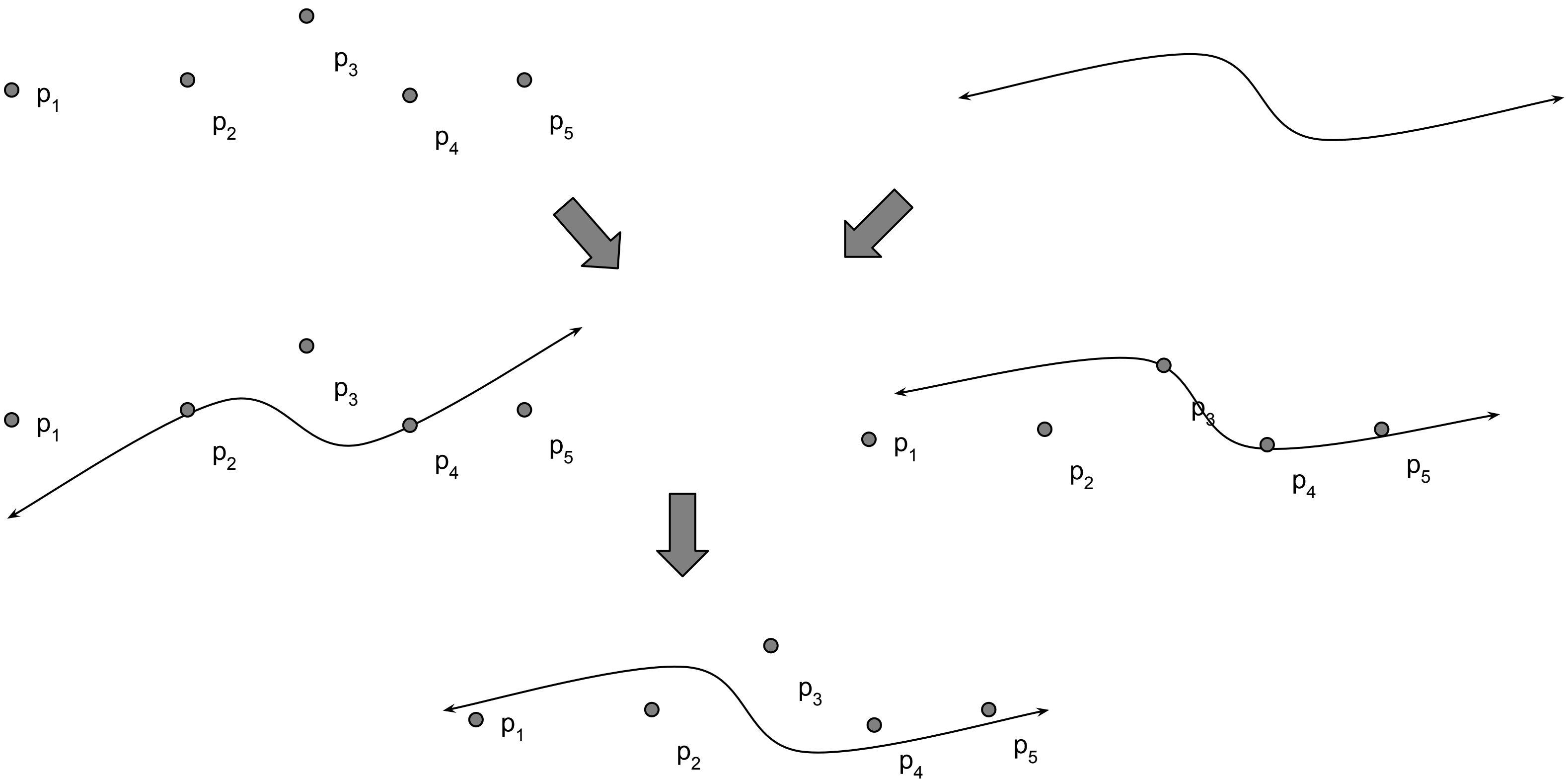


Approximation

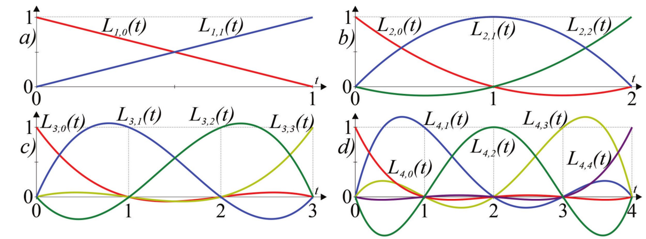
Approximation



Approximation



Classic interpolation approaches



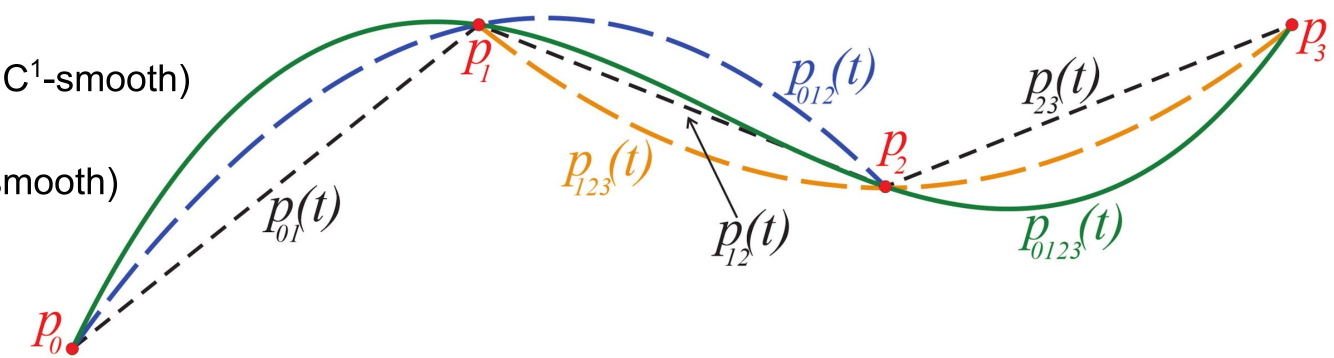
[LakYY] Figure 5.2: Lagrange Polynomials

Four points interpolated by Neville's Algorithm, [Gol03].

- Geometric interpretation of Newton/Lagrange polynomials
- Pyramidic algorithm

Resulting in the curves

- linear (1st-degree, C^0-smooth)
  - P_{01}, P_{12}, P_{23}
- quadratic (2nd-degree, C^1-smooth)
  - P_{012}, P_{123}
- cubic (3rd-degree, C^2-smooth)
  - P_{0123} -



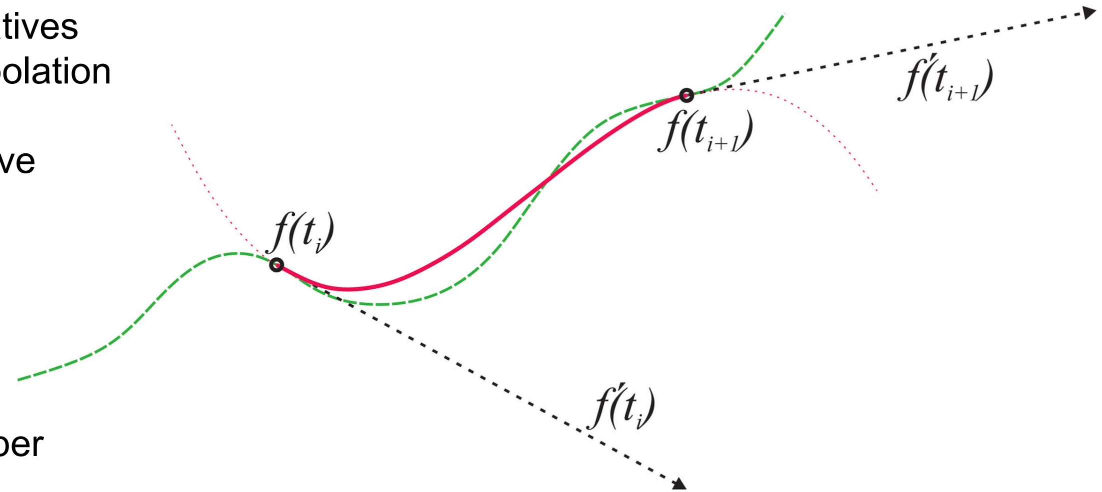
[LakYY] Figure 5.3: 1st, 2nd, and 3rd-degree curves

[Gol03] Pyramid Algorithms: A Dynamic Programming Approach to Curves and Surfaces for Geometric Modeling, R. Goldmann, 2003.
[LakYY] "Bending techniques in Curve and Surface constructions", Arne Lakså, Unpublished.

Hermite interpolation

Uses both points and associated derivatives

- Closely related to Newton's interpolation formula with divided differences
- Blends points along their respective derivative



[LakYY] Figure 5.4: 3rd-degree interpolation curve

Degree of the resulting function is number of data points and vectors - 1

- 2 points: $f(t_i)$ and $f(t_{i+1})$
- 2 derivatives: $f'(t_i)$ and $f(t_{i+1})$
- 3rd-degree

[LakYY] “Bending techniques in Curve and Surface constructions”, Arne Lakså, Unpublished.

SPLINES

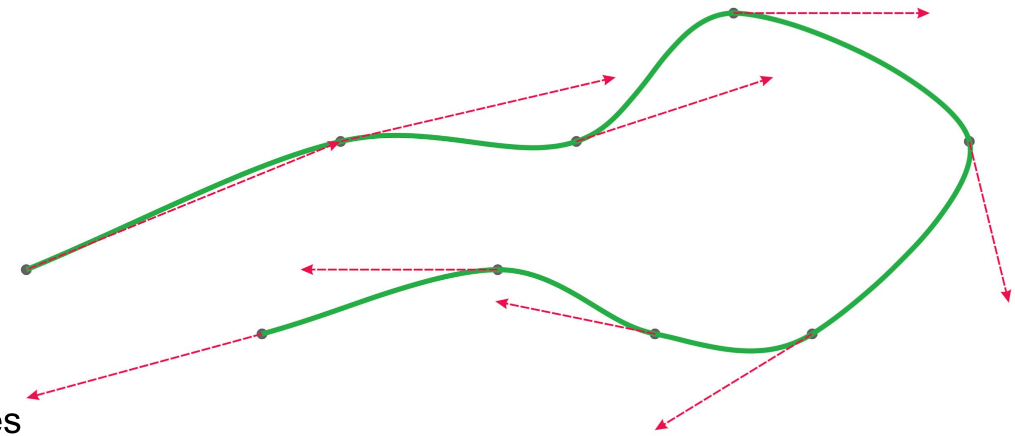
Splines (piecewise interpolation)

Polynomial functions of high degree can be non-beneficial

- High number of interpolation points
  =>
  High degree polynomial curve
- Runge's phenomenon, [Run01]

Stitch together adjacent lower degree curves

- Piecewise
- Degree and smoothness is a minimum over its parts and joints

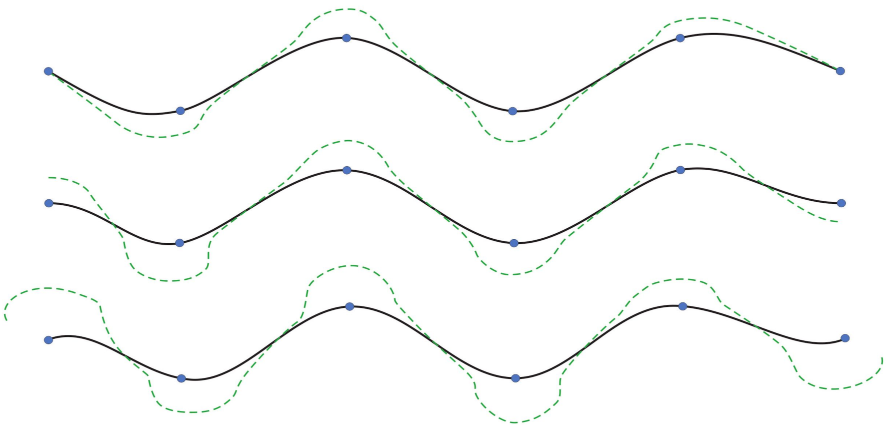


[LakYY] Figure 5.5: Catmull-Rom spline curve

[CR74] "A class of local interpolating splines", Catmull, Rom, 1974
[LakYY] "Bending techniques in Curve and Surface constructions", Arne Lakså, Unpublished.
[Run01] "Über empirische Funktionen und die Interpolation zwischen äquidistanten Ordinaten", Carl Runge, 1901

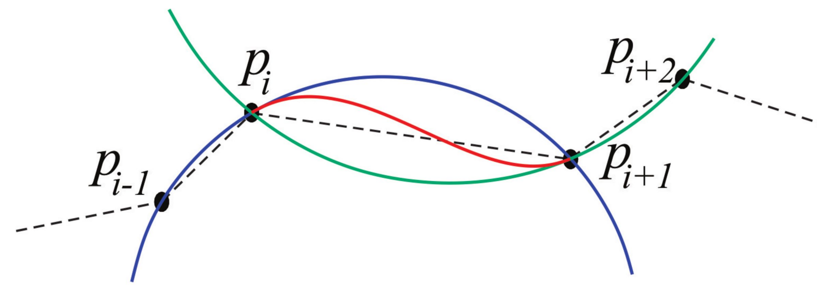
Two other examples: cubic- and circle-splines

Energy distribution



[LakYY] Figure 5.6 : cubic spline energy distribution

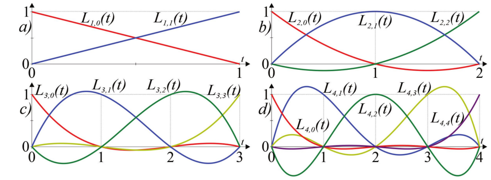
Special purpose construction



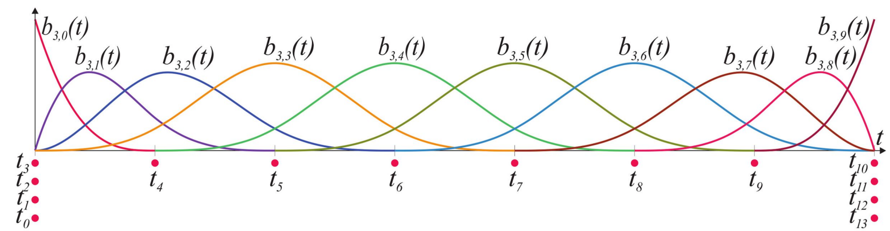
[LakYY] Figure 5.7 : circle spline

[deB78] "A Practical Guide to Splines", C. de Boor., Applied Mathematical Science, 1978
[LakYY] "Bending techniques in Curve and Surface constructions", Arne Laksâ, Unpublished.
[Sch81] "Spline Functions: Basic Theory", A Wiley-interscience publication, L. L. Schumaker, 1981
[Wen96] "Interpolation of curve data by blended generalized circles", H. Wenz, Computer Aided Geometric Design, 1996.

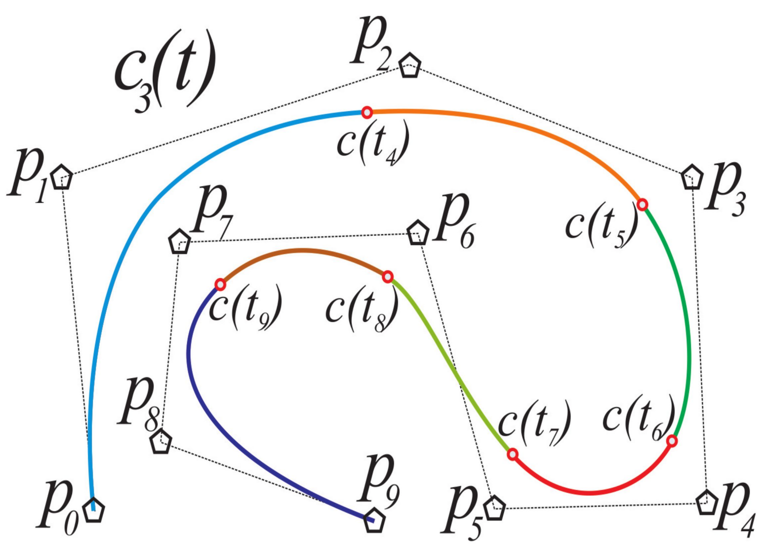
B-spline - the modern splines



[LakYY] Figure 5.2: Lagrange Polynomials



[LakYY] Figure 6.1



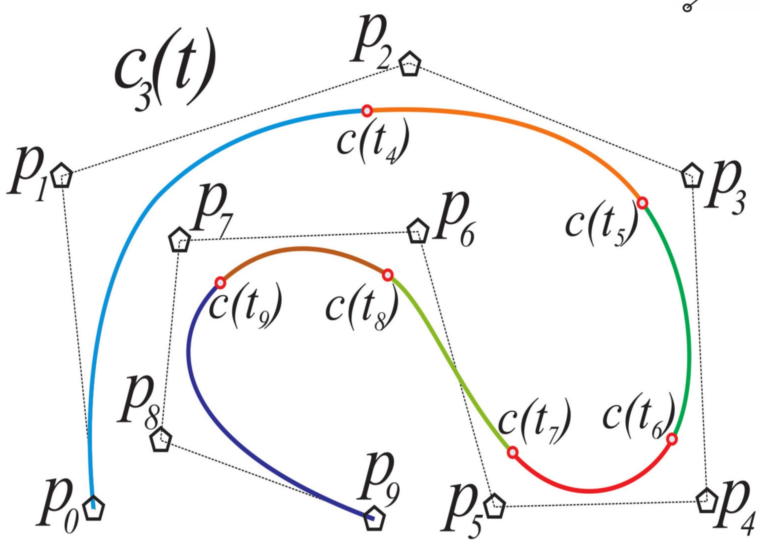
[LakYY] Figure 6.2

Some B-spline properties

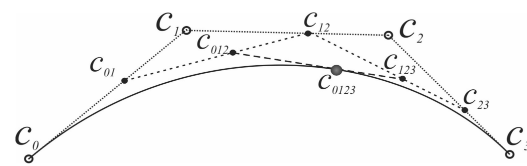
- Partition of Unity
- Affine invariance => stable under computation
- Clamped curve starts/ends in start/end of control polygon

[LakYY] “Bending techniques in Curve and Surface constructions”, Arne Lakså, Unpublished.

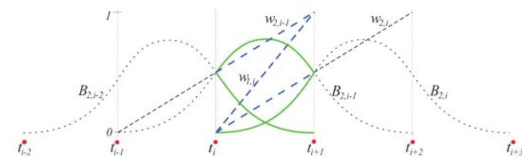
B-spline - the modern splines



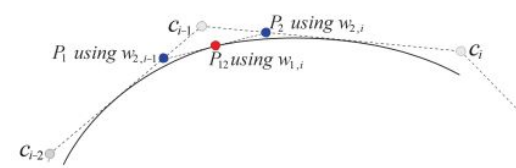
[LakYY] Figure 6.2



[LakYY] Figure 4.12



[LakYY] Figure 6.10



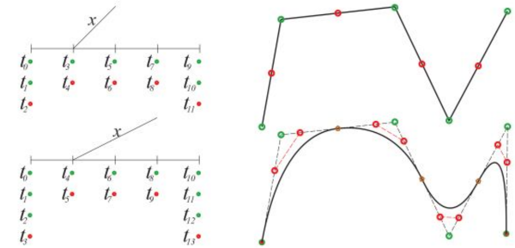
[LakYY] Figure 6.11

Some B-spline curve properties

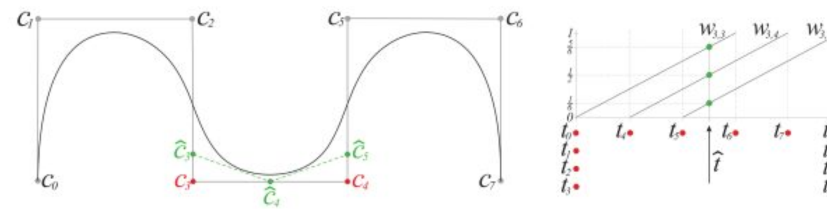
- Partition of Unity
- Affine invariance => stable under computation
- Clamped curve starts/ends in start/end of control polygon
- Generalization of the Bezier curve, which enables for instance de Casteljau's corner cutting algorithm

[LakYY] "Bending techniques in Curve and Surface constructions", Arne Laksâ, Unpublished.

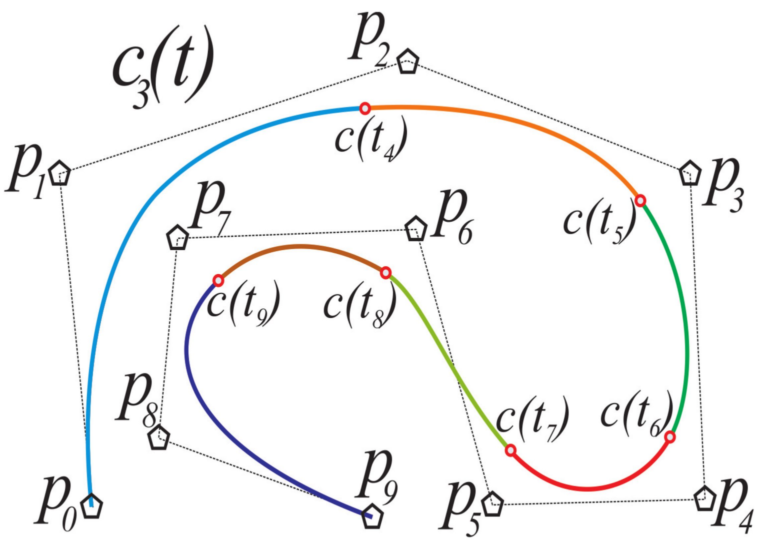
B-spline - the modern splines



[LakYY] Figure 6.16



[LakYY] Figure 6.15



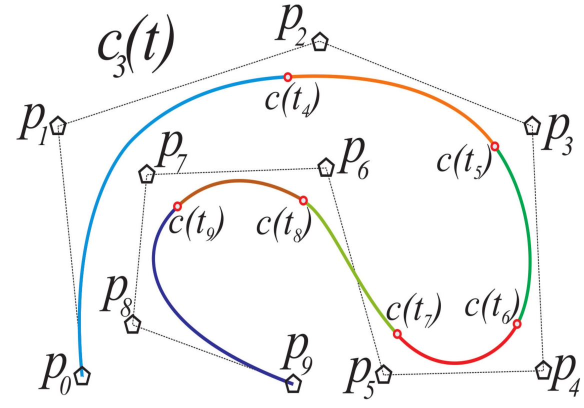
[LakYY] Figure 6.2

Some B-spline curve properties

- Partition of Unity
- Affine invariance => stable under computation
- Clamped curve starts/ends in start/end of control polygon
- Generalization of the Bezier curve, which enables for instance de Casteljau’s corner cutting algorithm
- Stable knot-insertion and degree-elevation

[Boe80] “Inserting New Knots into B-spline Curves”, Boehm, 1980
[CLR80] “Discrete B-splines and subdivision techniques in computer aided geometric design and computer graphics”, Cohen, Lyche, Riesenfeld, 1980.
[LakYY] “Bending techniques in Curve and Surface constructions”, Arne Lakså, Unpublished.

B-spline - fitting methods



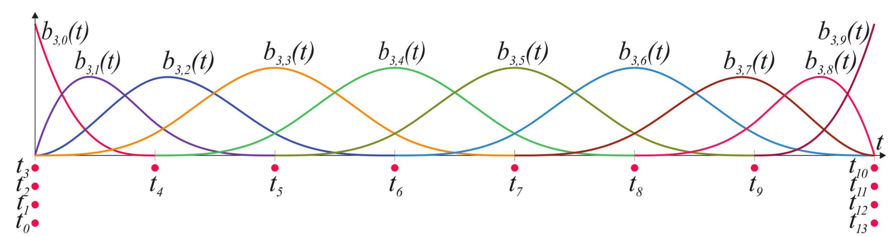
[LakYY] Figure 6.2

Global vs. local approximation methods

- Global methods => utilize all data points
- Local methods => utilize local supported points

Quasi-interpolation methods

- Tailored schema based methods => grows the local support



[LakYY] Figure 6.1

[LakYY] “Bending techniques in Curve and Surface constructions”, Arne Lakså, Unpublished.

Methods for interpolation

SMOOTH SPLINE SURFACES

Spline surface constructions

Classic constructions
- Cubic
- Hermite
- Circle
- B-spline

B-splines
- NURBS
- THB-spline (hierarchical)
- T-spline
- LR B-spline
- DCB-spline
- PHT-spline
- Blending
- Patchwork-splines
- Box-spline
- Others ...

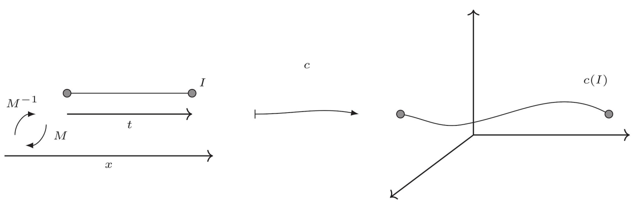
Spline-/patch-network
- Bezier
- Gregory
- Coons
- Blending (sub)
- Triangular
- Others ...

Blending constructions
- Circle
- GERBS (ERBS, BFBS, …)

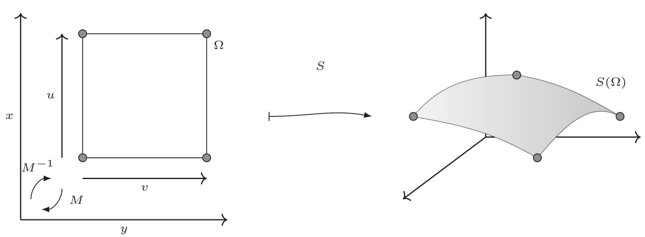
Other “categories” of “spline constructions”
- Developable
- Control Vectors
- Sub-division
- Others ...

Tensor product surfaces

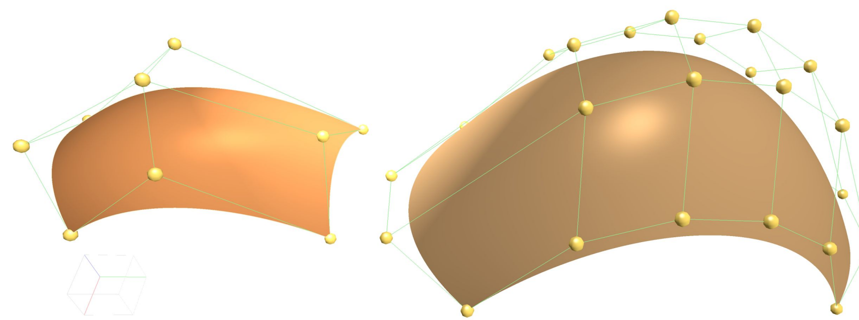
Schematic parametric **curve** mapping



Schematic parametric **surface** mapping



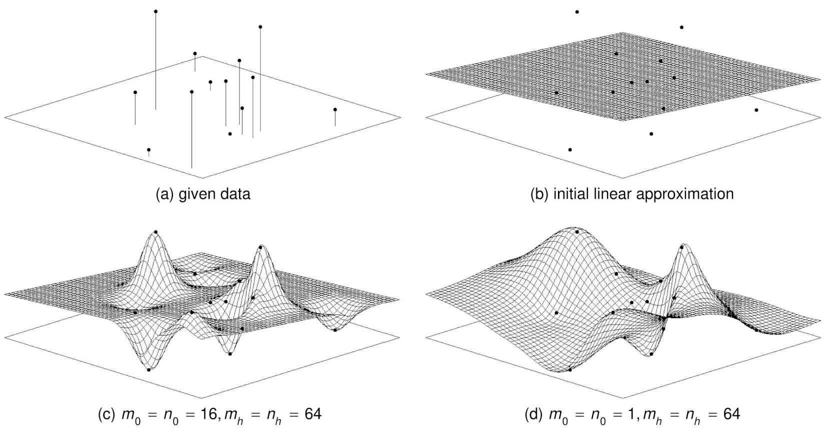
[LakYY] Figure 9.9



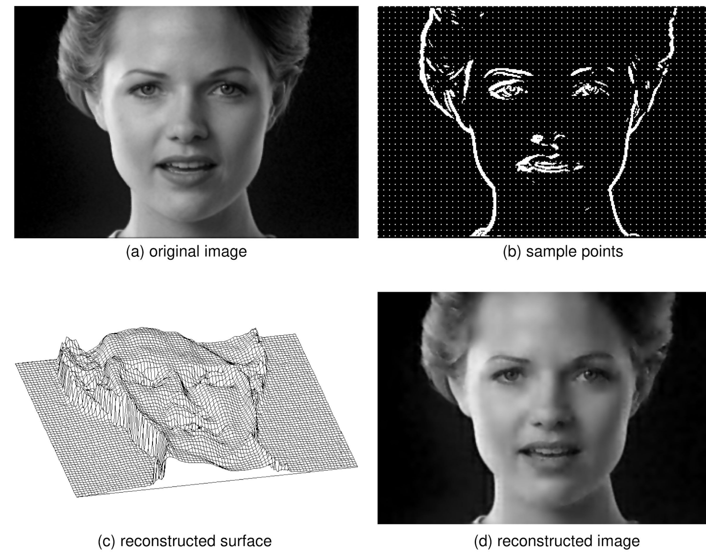
[LakYY] Figure 9.8

[LakYY] “Bending techniques in Curve and Surface constructions”, Arne Laksâ, Unpublished.

Smooth surfaces interpolation



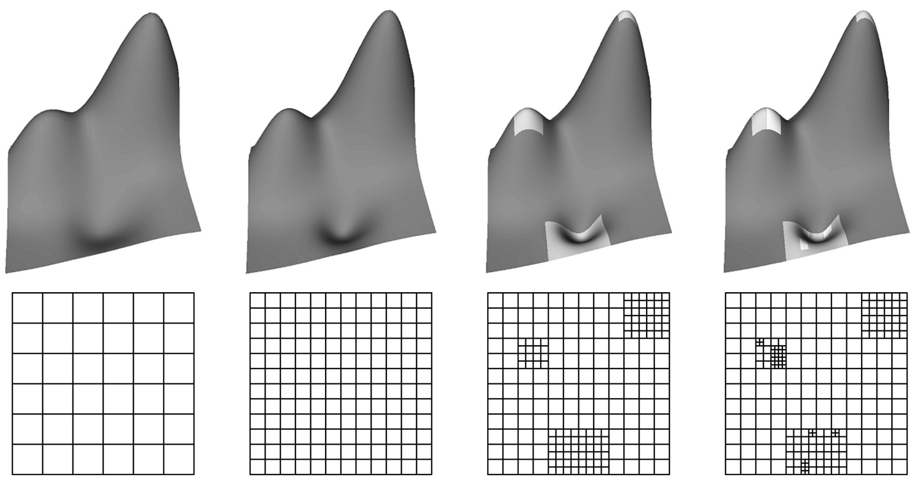
[LWS97] Figure 12



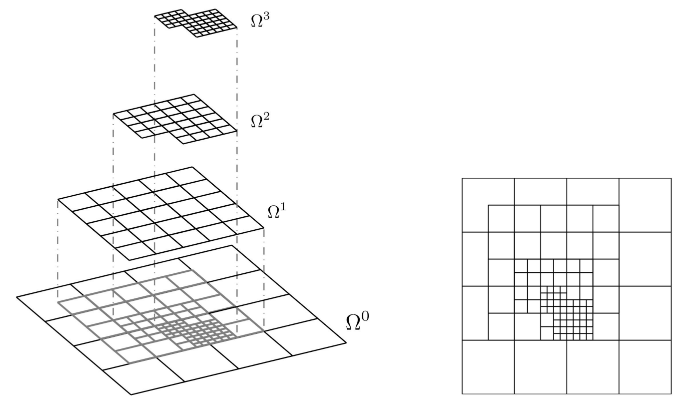
[LWS97] Figure 7

[LWS97] "Scattered data interpolation with multilevel b-splines", Lee, Wolberg, Shin, 1997

Hierarchical local refinement of TP



[GH97] Figure 6

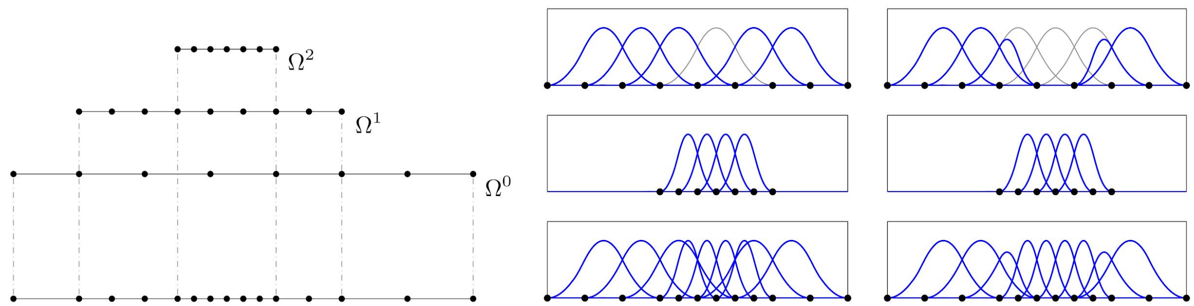


(a) nested domains (b) hierarchical mesh

[GSJ12] Figure 02

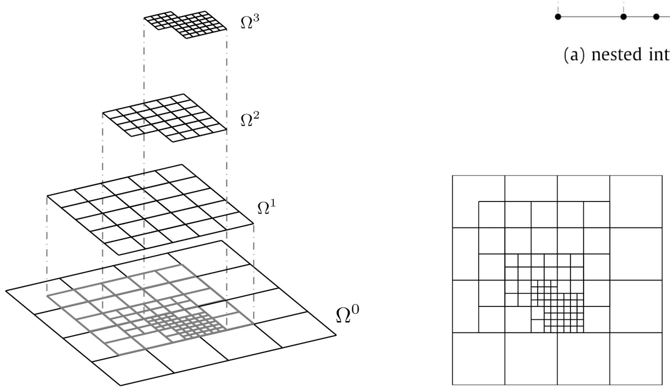
[GH97] "Interpolating and approximating scattered 3D-data with hierarchical tensor product B-splines", Greiner, Hormann, 1997
[GJS12] "THB-splines: the truncated basis for hierarchical splines", Giannelli, Juttler, Speleers, Computer Aided Geometric Design, 2012
[Kra97] "Hierarchical B-splines", Kraft, 1997
[Vu+11] "A hierarchical approach to adaptive local refinement in isogeometric analysis", Voung, Giannelli, Juttler, Simeon, CM Appl. Mech. Engrg., 2011

Hierarchical local refinement of TP



(a) nested intervals of level 0, 1, 2, and the hierarchical mesh (b) HB-splines of level 1 and 2 (c) THB-splines of level 1 and 2

[GSJ12] Figure 03



(a) nested domains (b) hierarchical mesh

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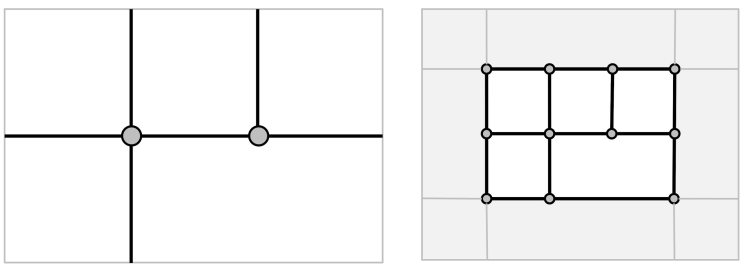
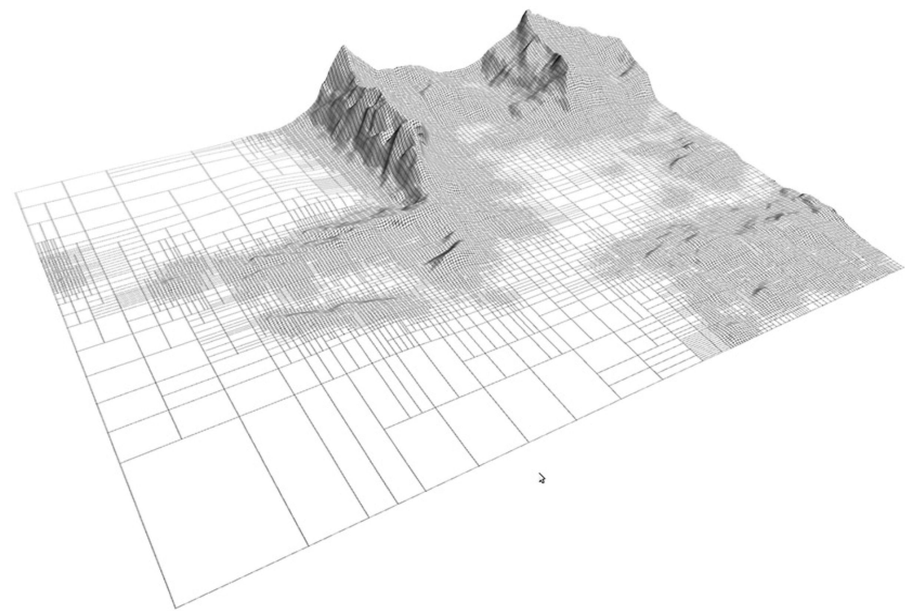
[Kra97] "Hierarchical B-splines", Kraft, 1997
[Vu+11] "A hierarchical approach to adaptive local refinement in isogeometric analysis", Voung, Giannelli, Juttler, Simeon, CM Appl. Mech. Engrg., 2011

Local refinement over box-partitions



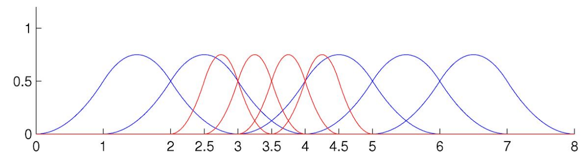
[Do+13] Figure 2



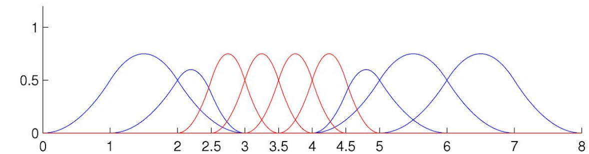
[Do+13] Figure 17

[Do+13] "Polygonal splines over locally refined box-partitions", Dokken, Lyche, Pettersen, 2013

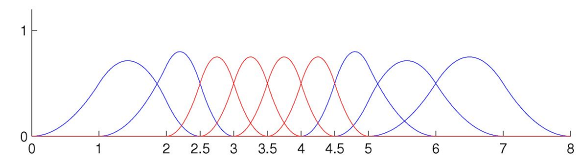
Local refinement over box-partitions



(a) Classical Hierarchical.

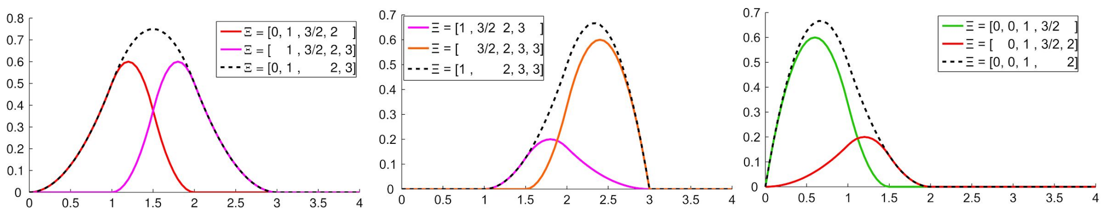


(b) Truncated Hierarchical.



(c) LR B-splines.

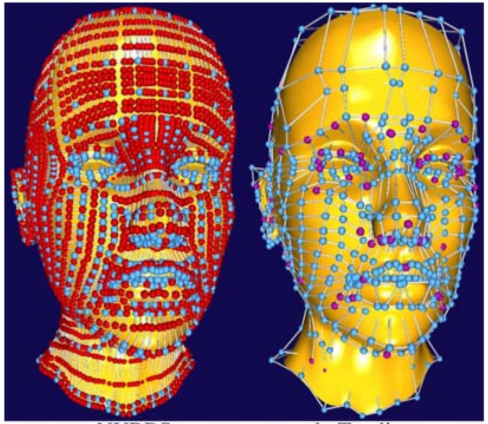
[JRK15] Figure 17



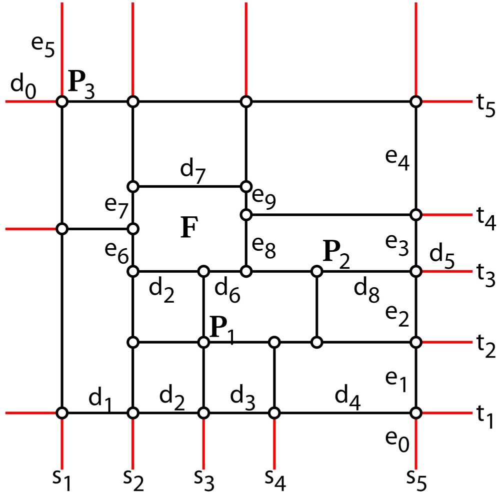
[JRK15] Figure 9

[JRK15] “On the similarities and differences between classical hierarchical truncated hierarch and lr b-splines,”, Johannessen, Remonato, Kvamsdal, 2015
[Pa+20] “Adaptive refinement with locally linearly independent LR B-splines: Theory and applications”, Patrizi, Manni, Pelosi, Speleers, 2020

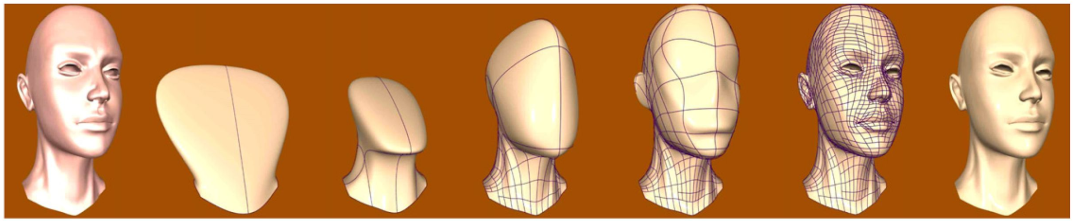
Local refinement over T-mesh



[SZL04] Figure 1



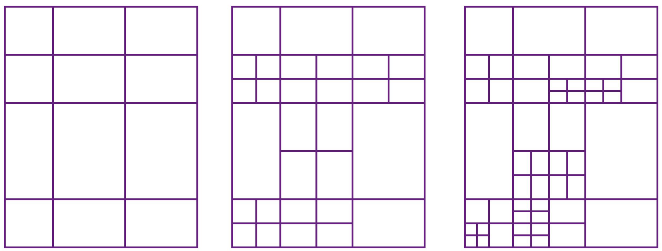
[Se+03] Figure 8



[De+08] Figure 1

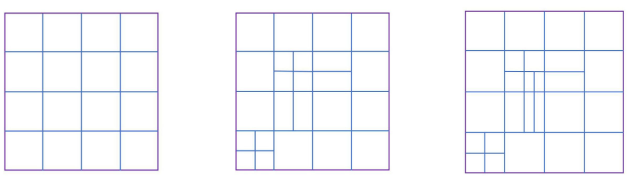
[De+08] "Polynomial splines over hierarchical T-meshes", Deng, Chen, Li, Hu, Tong, Yang, Feng, 2008
[Se+03] "T-splines and T-NURCCs", Sederberg, Zheng, Bakenov, Nasri, 2003
[SZL04] "T-spline Simplification and Local Refinement", Sederberg, Zheng, Lyche, 2004

Local refinement over T-mesh



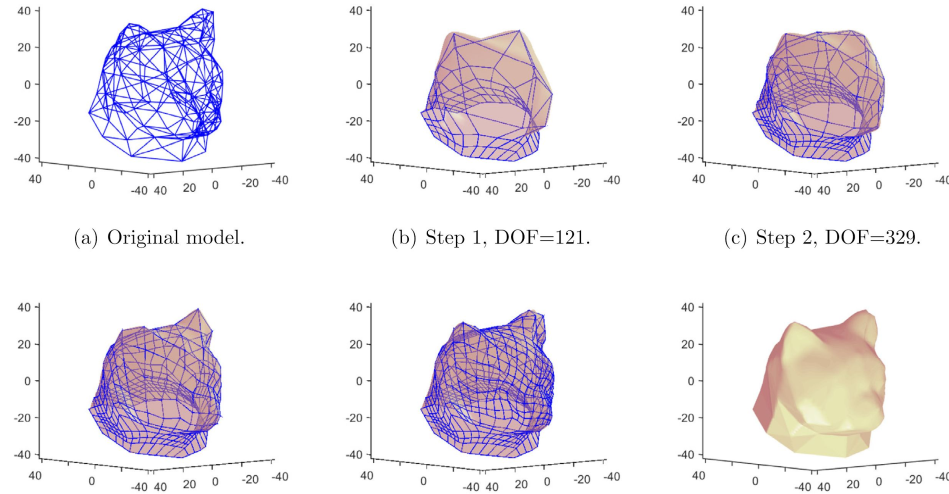
a. Level 0 b. Level 1 c. Level 2

[De+08] Figure 03



(a) $\mathbb{T}_0$ (b) $\mathbb{T}_1$ (c) $\mathbb{T}_2$

[NWD19] Figure 03



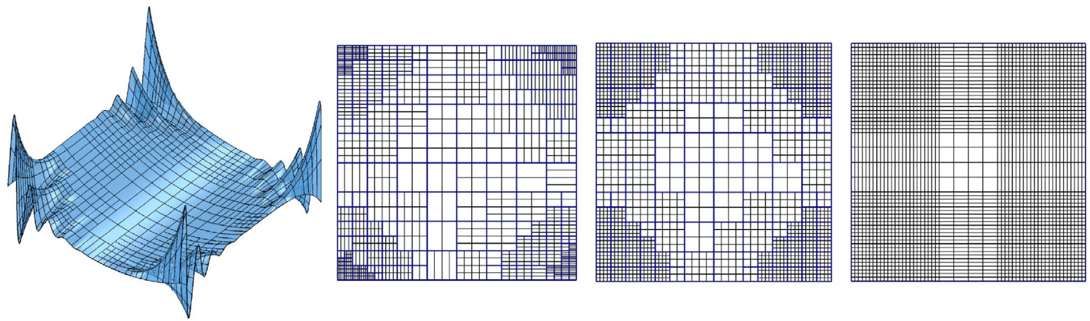
(a) Original model. (b) Step 1, DOF=121. (c) Step 2, DOF=329.

(d) Step 3, DOF=449 (e) Step 4, DOF=689. (f) Result surface.

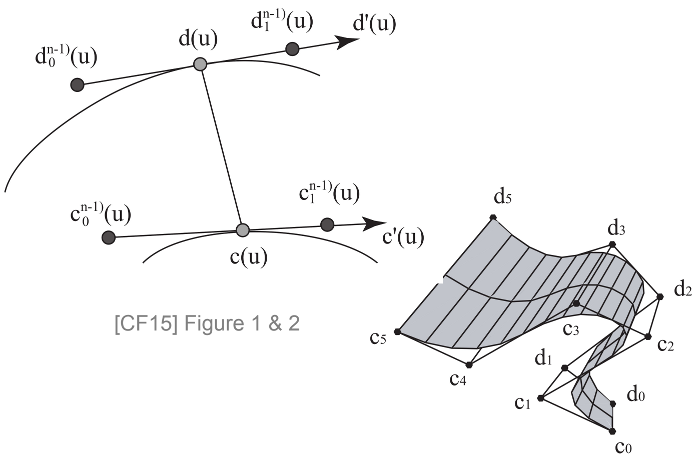
[ZH20] Figure 10

[De+08] “Polynomial splines over hierarchical T-meshes”, Deng, Chen, Li, Hu, Tong, Yang, Feng, 2008
[NWD19] “Modified PHT-splines”, Ni, Wang, Deng, Computer Aided Geometric Design, 2019
[Sc+12] “Local refinement of analysis-suitable T-splines”, Scott, Li, Sederberg, Hughes, 2012
[Zh+12] “On linear independence of T-spline blending functions”, Zheng, Sederberg, Hughes, Scott, 2012
[ZH20] “C^2 interpolation T-splines”, Zhu, Han, 2020

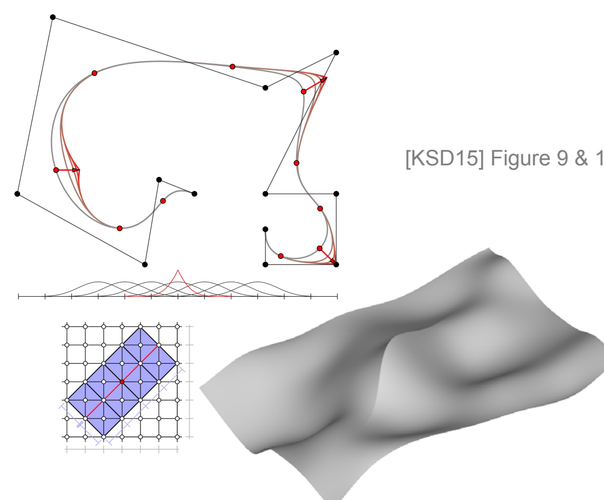
Developable, patchwork-splines, control vectors



[EJ17] Figure 8



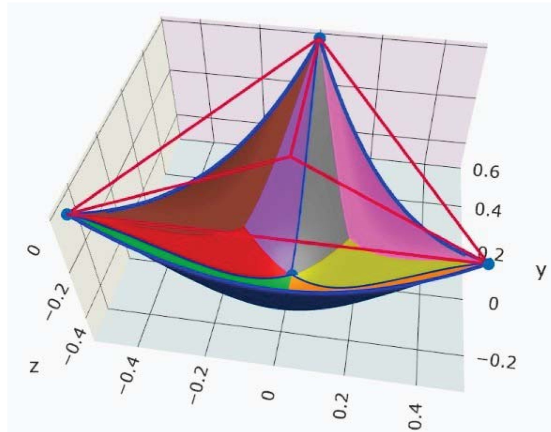
[CF15] Figure 1 & 2



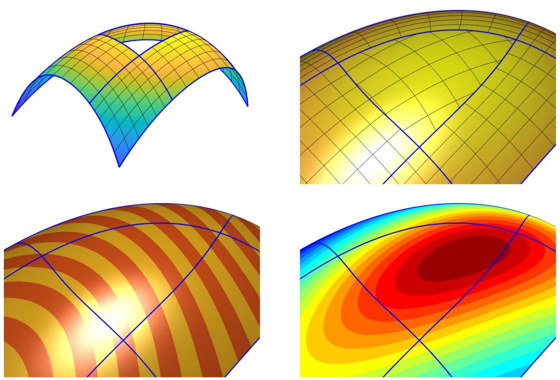
[KSD15] Figure 9 & 10

[Fer07] "B-spline control nets for developable surfaces", Fernandez-Jambrina, 2007
[CF15] "Interpolation of a spline developable surface between a curve and two rulings", Canton, Fernandez-Jambrina, 2015
[EJ17] "Patchwork B-spline refinement", Engleitner, Juttler, Computer-Aided Design, 2017
[EJ19] "Lofting with Patchwork B-Splines", Engleitner, Juttler, 2019 (book chapter)
[KSD15] "Control vectors for splines", Kosinka, Sabin, Dodgson, 2015

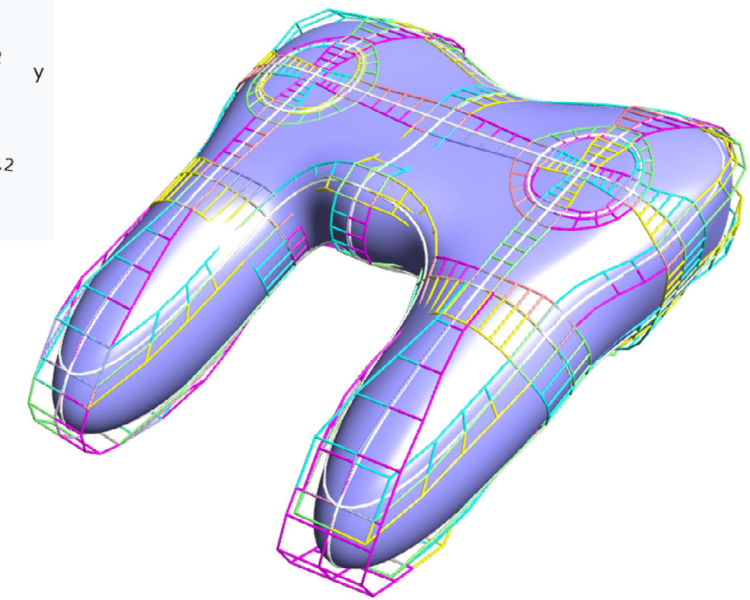
Spline networks, patches and patch-networks



[VR21] Figure 3b



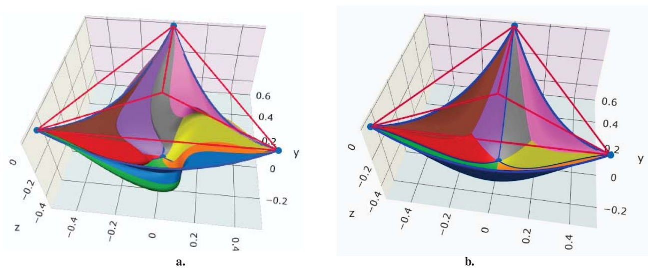
[MR21] Figure 11



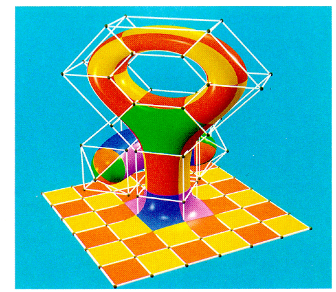
[VSK17] Figure 12a

[MR21] "Trimmed Spline Surfaces with Accurate Boundary Control", Martin, Reif, arXiv, 2021
[Nie83] "A Method for Interpolating Scattered DataBased Upon a Minimum Norm Network*, Nielson, 1983
[VR21] "Interpolation of data in $\mathbb{R}^3$ using quartic triangular Bézier surfaces", Vlachkova, Radev, 2021
[VSK17], "Enhancement of a multi-sided Bezier surface representation", Varady, Salvi, Kovacs, 2017

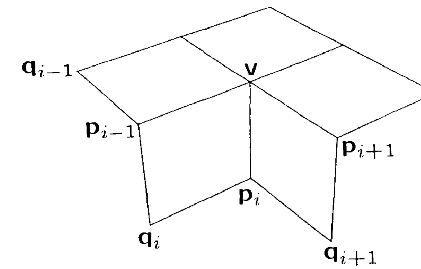
Patches in spline networks and patch-networks



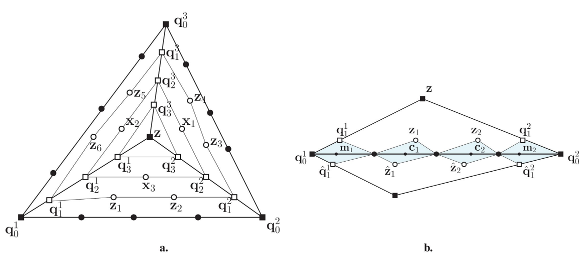
[VR21] Figure 3



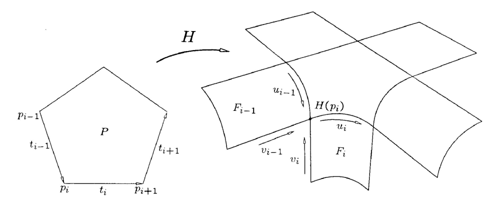
[LD90] Figure 11



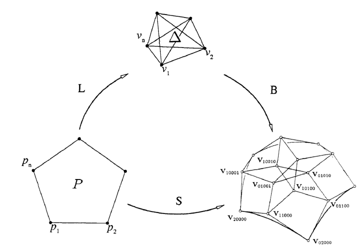
[LD90] Figure 04



[VR21] Figure 1



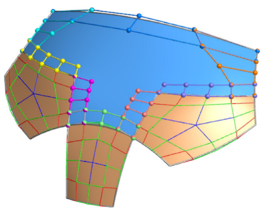
[LD90] Figure 02



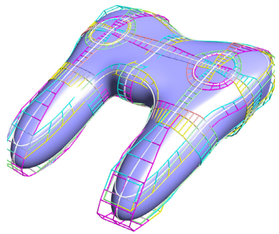
[LD90] Figure 01

[LD90] “Generalized B-spline Surfaces of Arbitrary Topology”, Loop, DeRose, 1990
[Vla21] “Interpolation of data in $\mathbb{R}^3$ using quartic triangular Bézier surfaces”, Vlachkova, Radev, 2021

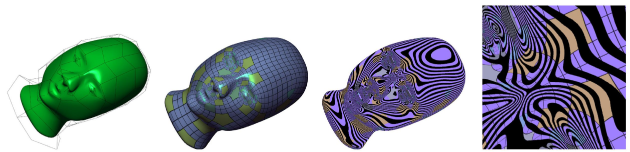
Patch-networks


(a) Control structures

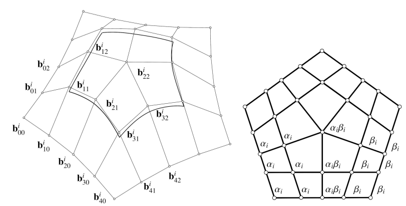
[SV18] Figure 22a



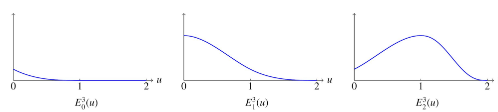
[VSK17] Figure 12a



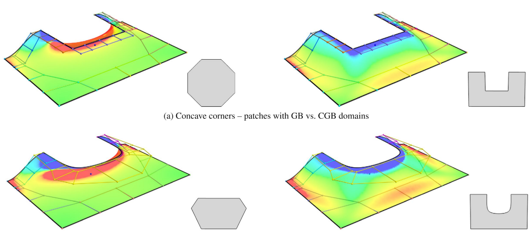
[HK20] Figure 6 (head model)



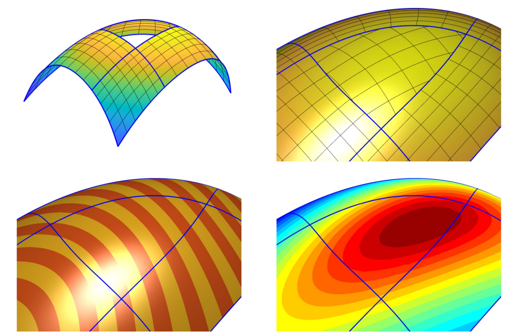
[HK20] Figure 2



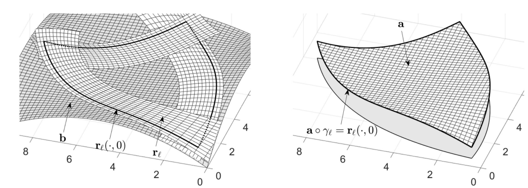
[HK20] Figure 3



[Va+20] Figure 01



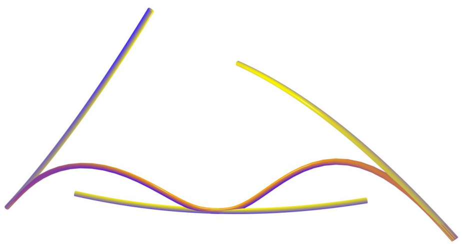
[MR21] Figure 11



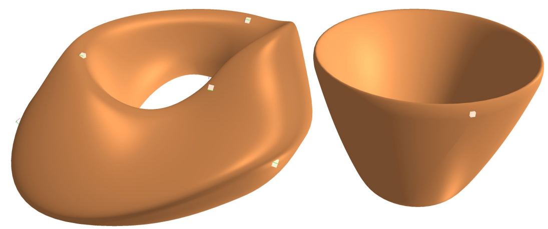
[MR21] Figure 2

[GC13] "An implicit G^1 multi patch B-spline interpolation for Kirchhoff–Love space rod", Greco, Cuomo 2013
[GC20] "An implicit G^1-conforming bi-cubic interpolation for the analysis of smooth and folded Kirchhoff-Love shell assemblies", Greco, Cuomo, 2020
[HK18] "Multisided generalisations of Gregory patches", Hettinga, Kosinka, 2018
[HK20] "A multisided C^2 B-spline patch over extraordinary vertices in quadrilateral meshes", Hettinga, Kosinka, 2020
[MR21] "Trimmed Spline Surfaces with Accurate Boundary Control", Martin, Reif, arXiv, 2021
[SV18] "Multi-sided Bezier surfaces over concave polygonal domains", Salvi, Varady, 2018
[Va+20] "Multi-sided Bezier surfaces over curved, multi-connected domains", Varady, Salvi, Vaitkus, Sipos, 2020
[VSK17], "Enhancement of a multi-sided Bezier surface representation", Varady, Salvi, Kovacs, 2017

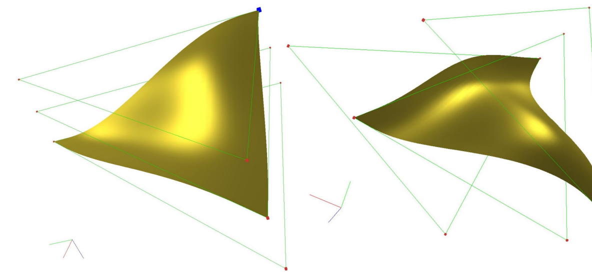
Blending splines (GERBS : ERBS, BFBS, ...)



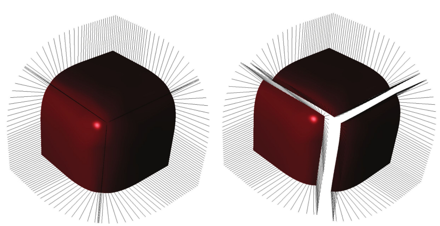
[Bra21] Figure 2.4



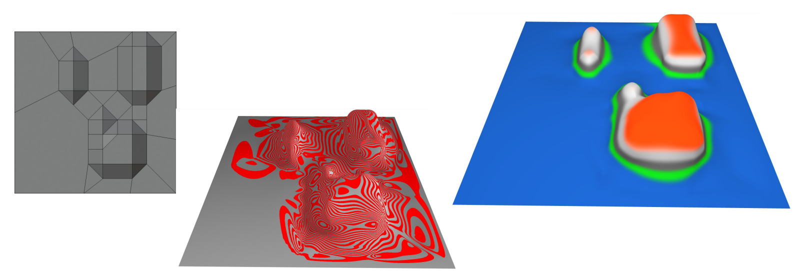
[LakYY] Figure 12.3



[LakYY] Figure 12.9



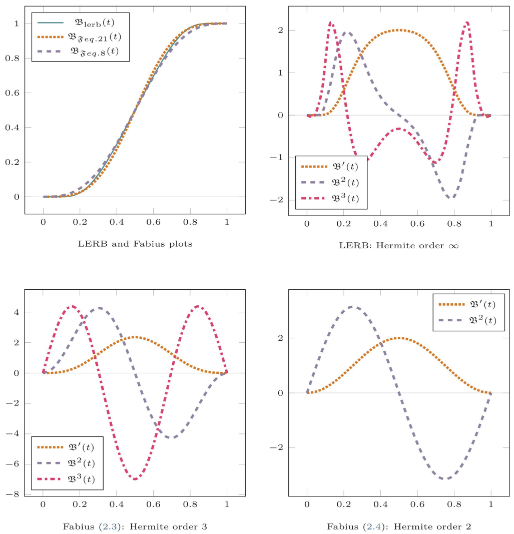
[LakYY] Figure 11.19



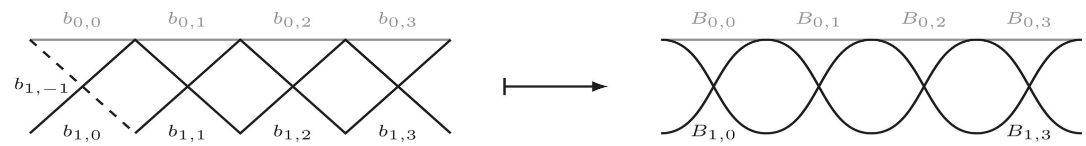
[Bra21] Figure 2.5

[Bra21] “Applications of blending splines in interactive geometric modeling”, Bratlie, Thesis
[DBL09] “Generalized Expo-Rational B-Splines”, Dechevsky, Bang, Lakså, IJPAM, 2009
[Lak13] “ERBS-surface construction on irregular grids”, Lakså, 2013
[LakYY] “Bending techniques in Curve and Surface constructions”, Arne Lakså, Unpublished.
[LBD05] “Exploring Expo-Rational B-splines for Curves and Surfaces”, Lakså, Bang, Dechevsky, 2005

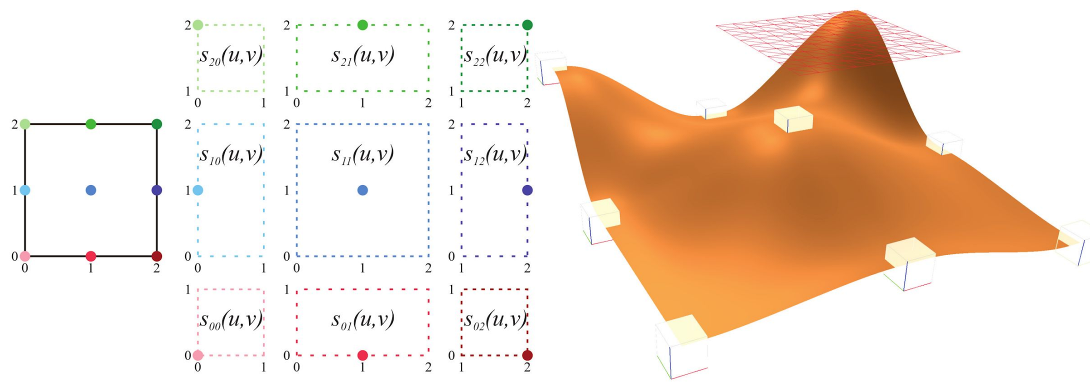
Blending splines - construction



[Bra21] Figure 2.2



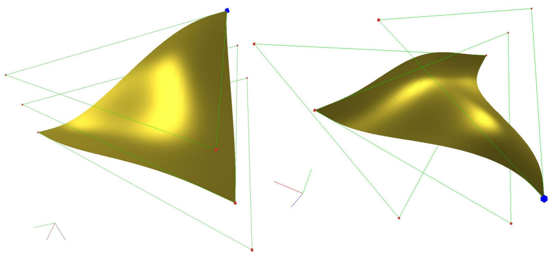
[Bra21] Figure 2.3



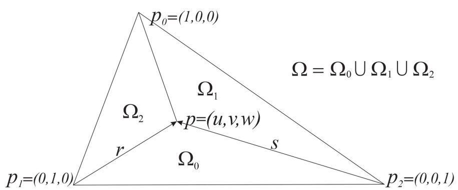
[LakYY] Figure 12.1

[Bra21] “Applications of blending splines in interactive geometric modeling”, Bratlie, Thesis
[DBL09] “Generalized Expo-Rational B-Splines”, Dechevsky, Bang, Lakså, IJPAM, 2009
[DLB06] “Expo-Rational B-Splines”, Dechevsky, Lakså, Bang, 2006
[DZ13] “Smooth GERBS, orthogonal systems and energy minimization”, Dechevsky, Zanaty, 2013
[Lak14] “Construction and properties of non-polynomial spline curves”, Lakså, 2014
[LBD05] “Exploring Expo-Rational B-splines for Curves and Surfaces”, Lakså, Bang, Dechevsky, 2005
[Olo19] “Blending functions based on trigonometric and polynomial approximations of the Fabius function”, Olofsen, 2019

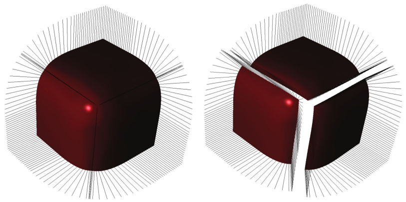
Blending splines - triangles, t-/star-joints, polygonal



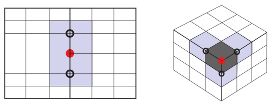
[LakYY] Figure 12.9



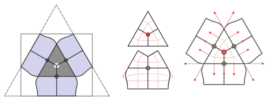
[LakYY] Figure 12.7



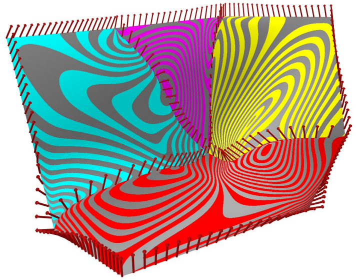
[LakYY] Figure 11.19



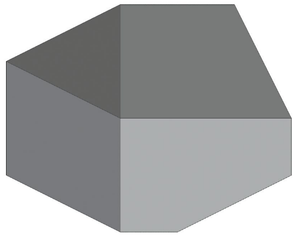
[LakYY] Figure 11.15



[LakYY] Figure 11.18



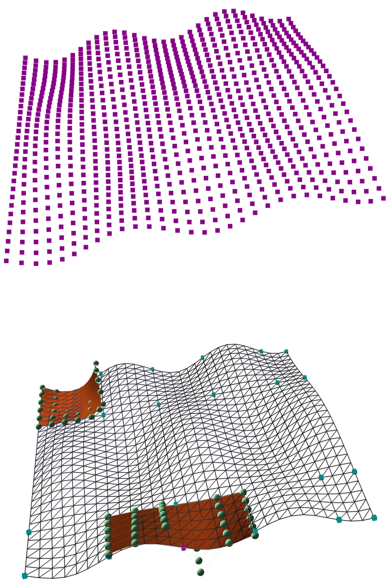
[Bra21] Figure V.16



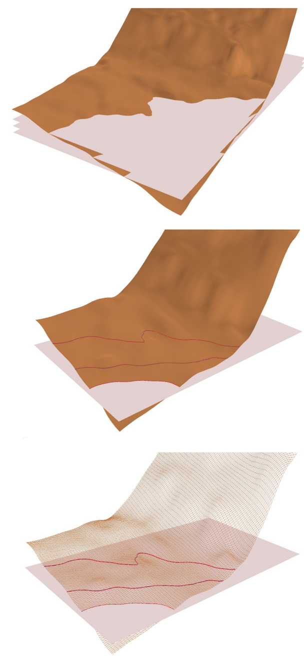
[Bra21] Figure V.14 Right

[Bra21] “Applications of blending splines in interactive geometric modeling”, Bratlie, Thesis
[Lak13] “ERBS-surface construction on irregular grids”, Lakså, 2013
[LakYY] “Bending techniques in Curve and Surface constructions”, Arne Lakså, Unpublished.

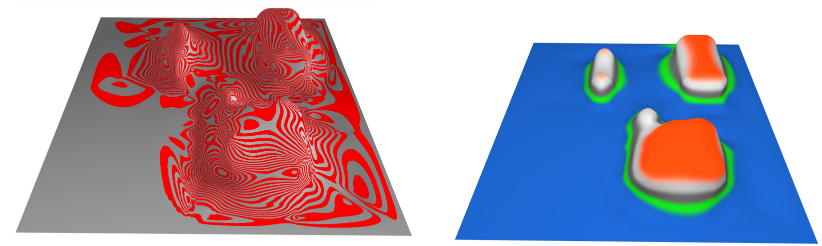
Blending splines - surface fitting



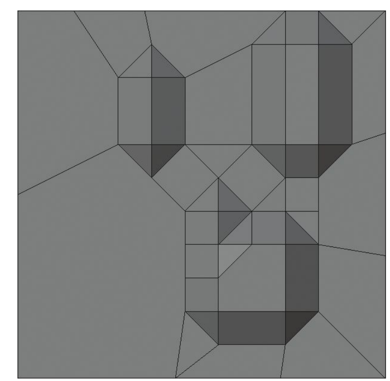
[DB15] Figure 2 & 3



[Ped20] Figure D.12



[Bra21] Figure 2.5



[Bra21] “Applications of blending splines in interactive geometric modeling”, Bratlie, Thesis
[DB15] “Data approximation using a blending type spline construction”, Dalmo, Bratlie, 2015
[Ped20] “Geometric modeling and sensor technology applications for engineering problems”, Pedersen, Thesis, 2020

Thank you for your attention ^^,